HEAT CONDUCTION OF ORTHOGONALLY REINFORCED COMPOSITE MATERIAL

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The problem of the heat flux in a medium with orthogonally positioned rows of periodically applied fibers is solved.

1. The thermophysical characteristics of so-called hybrid composite materials, as shown by the analysis of literature data, have only been determined by approximate methods to date. This is because the rigorous formulation of the heat-conduction problem is significantly three-dimensional even under the assumption of periodicity of the structure and long reinforcing elements. The solution of such problems by analytical or numerical methods is associated with considerable difficulties, in connection with the complexity of the region of temperature determination. Quantitative results of investigating the transverse heat conduction of an orthogonally reinforced composite fiber material are outlined below. A rigorous solution of the corresponding three-dimensional heat-conduction problem is used, allowing the structure of material of sufficiently general type to be considered.

2. Suppose that a composite material consists of a matrix and two types of fibers distributed as shown in Fig. 1. The coordinate systems introduced are related as follows: $z^{(2)} = z^{(1)} + h_3$, $x^{(2)} = -y^{(1)}$, $y^{(2)} = x^{(1)}$. It is assumed here that the longitudinal axes of fibers of the first series lie in the plane $z^{(1)} = 0$ parallel to the axis $x^{(1)}$, and the distance between two adjacent fibers is b. Fibers of the second series are parallel to the axis $x^{(2)}$, and their axes are in the plane $z^{(2)} = 0$ with a period a in the direction $y^{(2)}$. The whole composite material consists of such layers, i.e., its structure is periodic in the direction of the z axis with period h.

If the given material is in the field of a constant heat flux q_z, its temperature will be a periodic function with period a in the direction $x^{(1)}$ and b in the direction $x^{(2)}$, and quasi-periodic in the direction z (with period h).

3. Following [1], a layer of thickness containing two types of fiber rows is isolated. The corresponding heat-conduction boundary problem takes the form

$$\left(\frac{\partial^2}{\partial x^{(1)2}} + \frac{\partial^2}{\partial y^{(1)2}} + \frac{\partial^2}{\partial z^{(1)2}}\right)t = 0, \quad \begin{array}{l} -\infty < x^{(1)},\ y^{(1)} < \infty, \\ h_1 - h \leqslant z^{(1)} \leqslant h_1, \end{array} \tag{1}$$

with the boundary conditions

$$T\Big|_{\rho_k^{(j)}=R_j} = t_j\Big|_{\rho_k^{(j)}=R_j}, \quad \lambda_0 \frac{\partial}{\partial \rho_k^{(j)}} T\Big|_{\rho_k^{(j)}=R_j} = \lambda_j \frac{\partial}{\partial \rho_k^{(j)}} t_j\Big|_{\rho_k^{(j)}=R_j},$$
$$-\infty < k < \infty,$$
$$j = 1,\ 2.$$

The periodicity conditions on z are

$$T(x_k^{(j)},\ y_k^{(j)},\ z_k^{(j)} - h) = T(x_k^{(j)},\ y_k^{(j)},\ z_k^{(j)}) + \Gamma,$$
$$\frac{\partial}{\partial z_k^{(j)}} T(x_k^{(j)},\ y_k^{(j)},\ z_k^{(j)} - h) = \frac{\partial}{\partial z_k^{(j)}} T(x_k^{(j)},\ y_k^{(j)},\ z_k^{(j)}), \quad \begin{array}{l} -\infty < k < \infty, \\ j = 1,\ 2. \end{array} \tag{2}$$

Here $(\rho_k(j), \varphi_k(j))$ are cylindrical coordinates related to the k-th fiber of the j-th row; $t = T(x_k(j), y_k(j), z_k(j))$ in the region where $h_1 - h \leq z_k(j) \leq h_1$ and $\rho_k(j) \geq R_j$, while $t = t_{jk}(\rho_k(j), \varphi_k(j))$ in the region $\rho_k(j) \geq R_j$, j = 1, 2, 3; Γ is some constant. The coordinates $x_k(j)$, $y_k(j)$, $z_k(j)$, $\rho_k(j)$, j = 1, 2 are measured in fractions of R_1.

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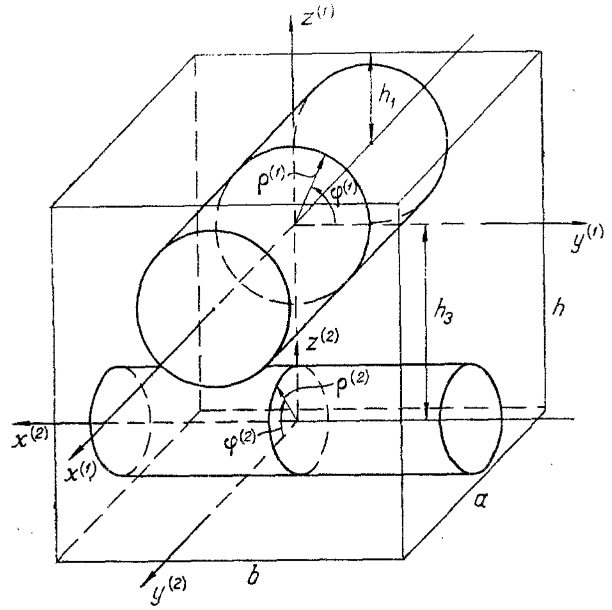

Fig. 1. Structural element.

The temperature distribution in all the fibers of the j-th row is the same; therefore, the matching conditions of the temperature fields are sufficiently well satisfied for fibers with k = 0.

Using the complete system of internal and external biperiodic solutions obtained in [2], the temperature fields $T(x^{(1)}, y^{(1)}, z^{(1)})$ in the matrix is written in the following form, taking account of the symmetry with respect to $x^{(1)}$ and $y^{(1)}$

$$T = A + Bz^{(1)} + \sum_{m,\,\mu=0}^{\infty}{}' D_{m\mu}^{(1)} U_{m\mu}^{(1)}(x^{(1)}, y^{(1)}, z^{(1)}) + \\ + \sum_{n,\mu=0}^{\infty}{}' D_{n\mu}^{(2)} U_{n\mu}^{(2)}(x^{(2)}, y^{(2)}, z^{(2)}) + \sum_{m,n=0}^{\infty}{}' (G_{mn}\exp[\delta_{mn}z^{(1)}] + H_{mn}\exp[-\delta_{mn}z^{(1)}])\cos(\alpha_m x^{(1)})\cos(\beta_n y^{(1)}), \tag{3}$$

where $\alpha_m = (2\pi/a)m$; $\beta_n = (2\pi/b)n$; $\delta_{mn} = \sqrt{\alpha_m^2 + \beta_n^2}$; A, B are constants. The prime denotes absence of the terms with superscripts simultaneously equal to zero in the sum. The diperiodic functions $U_{m\mu}^{(1)}$ and $U_{n\mu}^{(2)}$ take the form

$$U_{m\mu}^{(j)} = \frac{1}{2i^{\mu}} \sum_{q=-\infty}^{\infty} K_{\mu}(\varkappa_{m2}^{(j)}\rho_q^{(j)})[\exp(i\mu\varphi_q^{(j)}) + (-1)^{\mu}\exp(-i\mu\varphi_q^{(j)})]\cos(\varkappa_{m2}^{(j)}x^{(j)}),$$

$$U_{0\mu}^{(j)} = \frac{1}{2i^{\mu}} \sum_{q=-\infty}^{\infty} \left(\frac{1}{\rho_q^{(j)}}\right)^{\mu} \left[\exp(-i\mu\varphi_q^{(j)}) + (-1)^{\mu}\exp(i\mu\varphi_q^{(j)})\right],$$

$$\rho_q^{(j)} = \sqrt{y_q^{(j)2} + z_q^{(j)2}},\ \exp(i\varphi_q^{(j)}) = \frac{y_q^{(j)} + iz_q^{(j)}}{\rho_q^{(j)}},\quad y_q^{(j)} = y^{(j)} - qa^{(j)},\quad z_q^{(j)} = z^{(j)}, \tag{4}$$

$$U_{m\mu}^{(j)} = \begin{cases} \sum_{n=0}^{\infty} C_{mn\mu}^{(j)}\exp[-\delta_{mn}z^{(j)}]\cos(\varkappa_{n1}^{(j)}y^{(j)})\cos(\varkappa_{m2}^{(j)}x^{(j)}), & z^{(j)}>0, \\ (-1)^{\mu}\sum_{n=0}^{\infty} C_{mn\mu}^{(j)}\exp[\delta_{mn}z^{(j)}]\cos(\varkappa_{n1}^{(j)}y^{(j)})\cos(\varkappa_{m2}^{(j)}x^{(j)}), & z^{(j)}<0, \end{cases}$$

where

$$C_{mn\mu}^{(j)} = \frac{\pi}{a^{(j)}\delta_{mn}}\varepsilon_n\xi_{mn}^{\mu(j)};\quad C_{0n\mu}^{(j)} = \frac{2\pi}{a^{(j)}}\varepsilon_n\frac{\varkappa_{n1}^{(j)\mu-1}}{(\mu-1)!}(-1)^{\mu};$$

$$C_{001}^{(j)} = -\frac{\pi}{a^{(j)}};\quad \xi_{mn}^{\mu(j)} = \frac{1}{|\varkappa_{m2}^{(j)}|^{\mu}}[(\delta_{mn}+\varkappa_{n1}^{(j)})^{\mu} + (\delta_{mn}-\varkappa_{n1}^{(j)})^{\mu}],\ j=1,\ 2;$$

TABLE 1. Effective Thermal Conductivity of Fiber Medium for Various Geometric Parameters of the Structure

f	b	h_3	h	Thermal conductivity of fibers			
				$\frac{\lambda_1}{\lambda_0}=100$, $\frac{\lambda_2}{\lambda_0}=100$	$\frac{\lambda_1}{\lambda_0}=100$, $\frac{\lambda_2}{\lambda_0}=10$	$\frac{\lambda_1}{\lambda_0}=100$, $\frac{\lambda_2}{\lambda_0}=0$	$\frac{\lambda_1}{\lambda_0}=0$, $\frac{\lambda_2}{\lambda_0}=0$
0,60	2,44	2,10	4,30	4,60	3,77	0,51	0,26
0,60	2,10	2,80	5,00	3,25	2,90	0,28	0,15
0,45	3,24	2,10	4,30	3,42	2,90	0,77	0,42
0,45	2,32	2,20	6,00	2,28	2,13	0,52	0,29
0,45	2,10	3,00	6,65	2,07	1,96	0,34	0,19
0,30	4,88	2,10	4,30	2,33	2,07	0,94	0,58
0,30	2,10	4,88	10,0	1,53	1,48	0,44	0,25

$$a^{(1)}=b;\quad a^{(2)}=a;\quad \varkappa_{n1}^{(1)}=\varkappa_{n2}^{(2)}=\beta_n;\ \varkappa_{m1}^{(1)}=\varkappa_{m2}^{(2)}=\alpha_m;\ \varepsilon_0=0{,}5;\ \varepsilon_p=1\ (p\geqslant 1).$$

Satisfaction of the periodicity conditions in z in Eq. (2) leads to the following relation

$$\begin{gathered}\Gamma=-\left[Bh+2\left(D_{01}^{(1)}C_{001}^{(1)}+D_{01}^{(2)}C_{001}^{(2)}\right)\right],\\ \Delta_{mn}\{d_{mn}^{(1)(2)}+\exp[-\delta_{mn}h_3]\,d_{mn}^{(2)(2)}-\exp[\delta_{mn}(2h_1-h)]\,d_{mn}^{(1)(1)}-\\ -\exp[\delta_{mn}(2h_1-h+h_3)]\,d_{mn}^{(2)(1)}\}=H_{mn}-\exp[\delta_{mn}(2h_1-h)]\,G_{mn},\\ \Delta_{mn}\{d_{mn}^{(1)(1)}+\exp[\delta_{mn}h_3]\,d_{mn}^{(2)(1)}+\exp[-\delta_{mn}(2h_1-h)]\,d_{mn}^{(1)(2)}+\\ +\exp[-\delta_{mn}(2h_1-h+h_3)]\,d_{mn}^{(2)(2)}\}=\exp[-\delta_{mn}(2h_1-h)]\,H_{mn}+G_{mn},\end{gathered}\tag{5}$$

where

$$\Delta_{mn}=[\exp(\delta_{mn}h)-1]^{-1};\quad d_{mn}^{(j)(l)}=\sum_{\mu=0}^{\infty}(-1)^{\mu l}D_{m\mu}^{(j)}C_{mn\mu}^{(j)},\quad j,\ l=1,\ 2.$$

The temperature of the fibers in rows I and II of the isolated layer is written as follows

$$t_j=\sum_{p=-\infty}^{\infty}\left\{\sum_{m=-\infty}^{\infty}{}' Q_{pm}^{(j)}I_p\left(|\varkappa_{m2}^{(j)}|\rho^{(j)}\right)\exp[i\varkappa_{m2}^{(j)}x^{(j)}]+Q_{p0}^{(j)}\rho^{(j)|p|}\right\}\exp[ip\varphi^{(j)}],\quad j=1,\ 2,\tag{6}$$

where $I_p(x)$ is a modified p-th-order Bessel function.

Using the addition theorem for the external solutions of the Laplace equation, the expansions in [3], and Eq. (5), the solution for the matrix is written in the cylindrical coordinates $(\rho^{(j)}, \varphi^{(j)})$. From the matching conditions for the temperature fields in the matrix and the fibers in Eq. (6), eliminating the unknowns $Q_{pm}^{(j)}$ (j = 1, 2), a closed infinite system of linear algebraic equations with the unknowns $D_{m\mu}^{(j)}$ (j = 1, 2) is obtained

$$\begin{gathered}-BR_j^{2p}\delta_p^1+D_{0p}^{(j)}\frac{\left(\dfrac{\lambda_j}{\lambda_0}+1\right)}{\left(\dfrac{\lambda_j}{\lambda_0}-1\right)}+\sum_{\mu=1}^{\infty}D_{0\mu}^{(j)}c_{p\mu}^{0(j)}+\sum_{n=1}^{\infty}\sum_{\mu=0}^{\infty}E_{n\mu}^{(j)}\gamma_{p\mu}^{0n(j)}=0,\\ p=\overline{1,\ \infty};\end{gathered}\tag{7}$$

$$D_{mp}^{(j)}q_{mp}^{(j)}+\sum_{\mu=0}^{\infty}D_{m\mu}^{(j)}c_{p\mu}^{m(j)}+\sum_{n,\mu=0}^{\infty}{}' E_{n\mu}^{(j)}\gamma_{p\mu}^{mn(j)}=0,\ m=\overline{1,\ \infty},\ p=\overline{0,\ \infty},\ j=1,\ 2,$$

where

$$q_{np}^{(j)}=\frac{1}{\left(1-\dfrac{\lambda_j}{\lambda_0}\right)}\left[\frac{\Theta_p^{(1)}(\varkappa_{n1}^{(j)}R_j)}{\varepsilon_p\Theta_p^{(2)}(\varkappa_{n1}^{(j)}R_j)}-\frac{\lambda_jK_p(\varkappa_{n1}^{(j)}R_j)}{\lambda_0\varepsilon_pI_p(\varkappa_{n1}^{(j)}R_j)}\right];$$

$$\Theta_p^{(1)}(z)=\frac{p}{z}K_p(z)-K_{p+1}(z);\quad \Theta_p^{(2)}(z)=\frac{p}{z}I_p(z)+I_{p+1}(z);$$

TABLE 2. Effective Thermal Conductivities of Fiber Medium with Various Filler Properties

$\frac{\lambda_1}{\lambda_0}$	$\frac{\lambda_2}{\lambda_0}$	Filler concentration					
		f_1	f_2	f_1	f_2	f_1	f_2
		0,46	0,12	0,35	0,09	0,29	0,07
0	0	0,17		0,21		0,25	
10	0	1,27		1,17		1,13	
100	0	1,37		1,24		1,18	
0	10	0,18		0,23		0,27	
10	10	2,94		1,97		1,66	
100	10	3,58		2,16		1,76	
0	100	0,19		0,23		0,27	
10	100	3,25		2,06		1,71	
100	100	4,08		2,26		1,82	

$$E_{n\mu}^{(1)} = D_{n\mu}^{(2)};\ E_{n\mu}^{(2)} = D_{n\mu}^{(1)};$$

$K_p(x)$ is a p-th-order MacDonald function; δ_p^1 is a Kronecker delta. The expressions for the coefficients $c_{p\mu}{}^{o}(j)$, $c_{p\mu}{}^{m}(j)$, $\gamma_{p\mu}{}^{on}(j)$, $\gamma_{p\mu}{}^{mn}(j)$ are not given here because of ther unwieldiness.

This infinite system may be reduced to a system of normal type under the condition of no tangency of the fiber surfaces, and hence admits of solution by the reduction method.

In computer realization of this problem, the unknowns with indices up to m, n, p, $\mu = 4$ are retained in the infinite system of unknowns in the calculations. This ensures high accuracy of the solution; the maximum error in meeting the thermal-contact conditions in Eq. (1) is of the order of 1%.

5. Calculating the effective heat-conduction characteristics entails averaging the derivatives of the solution with respect to the coordinates in the volume of the elementary cell (Fig. 1), including fibers of both series.

For the given problem $\left\langle \frac{\partial T}{\partial x} \right\rangle = \left\langle \frac{\partial T}{\partial y} \right\rangle = 0.$

Therefore

$$-\langle q_z \rangle = \lambda_{ef} \left\langle \frac{\partial T}{\partial z} \right\rangle. \tag{8}$$

The mean values of $\partial T/\partial z$ and q_z are written in the form

$$\left\langle \frac{\partial T}{\partial z} \right\rangle abh = \iiint_{V_M} \frac{\partial T}{\partial z} dV_M + \iiint_{V_1} \frac{dt_1}{\partial z} dV_1 + \iiint_{V_2} \frac{\partial t_2}{\partial z} dV_2,$$

$$-\langle q_z \rangle abh = \lambda_0 \iiint_{V_M} \frac{\partial T}{\partial z} dV_M + \lambda_1 \iiint_{V_1} \frac{\partial t_1}{\partial z} dV_1 + \lambda_2 \iiint_{V_2} \frac{\partial t_2}{\partial z} dV_2, \tag{9}$$

$$abh = V_M + V_1 + V_2,\quad V_1 = \pi R_1^2 a,\quad V_2 = \pi R_2^2 b.$$

Using the Ostrogradskii–Gauss formula and also Eqs. (2), (3), and (6) and the condition that the temperatures at the matrix and fiber boundaries are equal, the following result is obtained after appropriate transformations

$$\left\langle \frac{\partial T}{\partial z} \right\rangle = -\frac{1}{h}\Gamma,$$

$$-\langle q_z \rangle = \lambda_0 \left\{ -\frac{1}{h}\Gamma + 2\left[D_{01}^{(1)} \frac{f_1}{R_1^2} + D_{01}^{(2)} \frac{f_2}{R_2^2} \right] \right\}, \tag{10}$$

where $f_j = \pi R_j^2/a(j)h$, $j = 1, 2$. Hence the effective thermal conductivity is written as the ratio of $-\langle q_z \rangle$ to $\langle \partial T/\partial z \rangle$

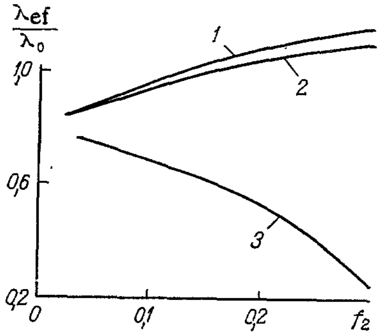

Fig. 2

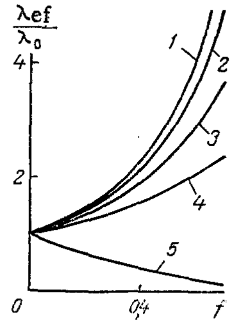

Fig. 3

Fig. 2. Dependence of $\lambda ef/\lambda_0$ on the fiber concentration f_z of one of the rows, in the presence of a row of pores in the orthogonal direction (λ_1/λ_0 = 0, f_1 = 0.1): 1) λ_2/λ_0 = 100; 2) 10; 3) 0.

Fig. 3. Curves of $\lambda ef/\lambda_0$ as a function of the fiber concentration f: 1) $\lambda j/\lambda_0$ = 700 (j = 1.2); 2) 40; 3) 10; 4) 4; 5) 0.

$$\lambda_{ef}=\lambda_0\left\{1-\frac{2h}{\Gamma}\left[D_{01}^{(1)}\frac{f_1}{R_1^2}+D_{01}^{(2)}\frac{f_2}{R_2^2}\right]\right\}. \tag{11}$$

Thus, to determine the effective thermal conductivity, it is sufficient to have the values of only the first two unknowns of the infinite system in Eq. (7).

6. The given problem includes a sufficiently large number of parameters. In fact, the structure of the composite material is determined by such geometric parameters as the fiber radii R_1, R_2; the fiber-packing periods in the rows a, b; the distance between the two adjacent rows chosen h_3; and the thickness of the characteristic layer h, which is the distance between the closest rows of fibers in the same direction. In addition, the thermal properties of the material are specified by three thermal conductivities λ_0, λ_1, λ_2. Thus, introducing dimensionless quantities, seven different parameters are obtained, in overall complexity; this leads to definite difficulties in choosing their numerical values for the calculations. Accordingly, the following considerations are appropriate. First, the parameters of the problem are changed so that the influence of the structure parameters and the difference in thermal conductivity of the individual phases on the effective heat conduction of the given composite material are established by means of a computational experiment. Second, a more uniform distribution of the filler in the matrix ($R_2 = R_1$, a = b, $h = 2h_3$, $\lambda_1 = \lambda_2$) is adopted, for comparison of the calculated values of the transverse thermal conductivity with those determined in [4].

Tables 1 and 2 give values of the relative effective thermal conductivity $\lambda ef/\lambda_0$ for different ratios of the structure parameters and thermal conductivities of the fibers (zero thermal conductivity corresponds to a pore). The data of Table 1 correspond to a = b, $R_2 = R_1$ and those of Table 2 to $R_2 = 0.5R_1$, $a = b = 2.1R_1$, $h = 2h_3$; h_3 takes the values $1.6R_1$, $2.1R_1$, $2.6R_1$ with a total concentration of 0.58, 0.44, 0.36, respectively. As follows from Table 1, the thermal conductivity of the composite with the same total concentration of fibers and fixed fiber thermal conductivity depends very significantly on the values of the structural parameters. Table 2 gives the variation in effective thermal conductivity as a function of the geometric and thermophysical parameters of the material, with strongly different radii of adjacent fibers (pores). Comparison of the data in Tables 1 and 2 leads to the conclusion that, with identical thermal conductivities and concentrations of filler, the thermal conductivity of the composite may depend strongly on the ratio of the radii R_1 and R_2.

The curves in Fig. 2 show that, in the presence of some fixed pore concentration, the fiber concentration in the orthogonal direction may increase, giving various values of the effective thermal conductivity of the composite material, including $\lambda ef = \lambda_0$ (curves 1 and 2). In this case, the structural parameters are chosen as follows: $R_2 = R_1$, $b = 6R_1$, $h_3 = 2.5R_1$, $h = 5R_1$, $\lambda_1 = \lambda_2$. Curve 3 corresponds to a material containing only pores ($\lambda j/\lambda_0$ = 0, j = 1.2).

The dependence of $\lambda ef/\lambda_0$ on the concentration $f = f_1 + f_2$ of fibers is shown in Fig. 3 for various values of the relative thermal conductivity $\lambda j/\lambda_0$ (j = 1, 2); curve 5 corresponds

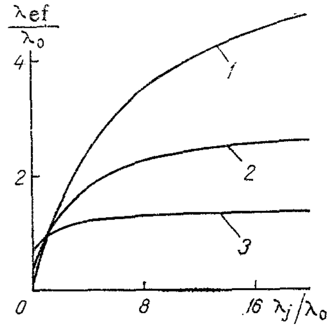

Fig. 4. Dependence of λ_{ef}/λ_0 on the thermal conductivities of the fibers λ_j/λ_0 ($j = 1, 2$): 1) f = 0.71; 2) 0.50; 3) 0.20.

to porous material. The dependence of λ_{ef}/λ_0 on λ_j/λ_0 ($j = 1, 2$) is shown in Fig. 4 for various values of the fiber concentration, with $R_2 = R_1$, $a = b$, $h = 2h_3$, $h_3 = a$, $\lambda_1 = \lambda_2$. As is evident from the figures, λ_{ef}/λ_0 increases at different rates with increase in f and the relative thermal conductivity λ_j/λ_0 ($j = 1, 2$), depending on the structural parameters.

Comparing the results obtained with the results of calculating the relative transverse thermal conductivity in [4], it may be noted that, at small concentrations (up to f = 0.4) and small values of the relative thermal conductivity of the fibers, the values of λ_{ef}/λ_0 are in good agreement; the discrepancy is from 0.5 to 4% with increase in λ_j/λ_0 ($j = 1, 2$). The agreement improves with increase in fiber concentration and greater closeness of the thermal conductivities to λ_0. Thus, for example, for $\lambda_j/\lambda_0 = 2$ ($j = 1, 2$), the calculated results for λ_{ef}/λ_0 coincide with those in [4] up to f = 0.6. With increase in λ_j/λ_0 ($j = 1, 2$), the discrepancy increases, and there is coincidence only at small concentrations. For f = 0.6 and $\lambda_j/\lambda_0 = 8$ ($j = 1, 2$), the difference is ~14%. More satisfactory agreement of the results is observed when $\lambda_j/\lambda_0 < 1$ ($j = 1, 2$).

Thus, the accurate solution of the heat-conduction problem constructed here for a composite medium with orthogonal packing of the fibers allows the effective heat conduction of the medium to be investigated over a wide range of variation of the geometric and thermophysical parameters. The strong dependence of the effective heat conduction on these parameters which is found indicates the possibility of controlling the heat conduction of composites of this class by changing their structure with a fixed concentration and heat conduction of the filler.

NOTATION

x, y, z, Cartesian coordinates; $\rho^{(j)}$, $\varphi^{(j)}$, cylindrical coordinates in a coordinate system associated with the j-th row of fibers; a, distance between neighboring fibers of the second row; b, distance between neighboring fibers f of the first row; h, thickness of the isolated layer containing two types of orthogonal fiber rows; h_1, distance between boundary of the isolated layer and a plane containing the axes of the fibers of the first row; h_3, distance between the axes of fibers in a different direction; R_j, fiber radius in j-th row; f_j, concentration of fibers in j-th row; λ_0, thermal conductivity of matrix material; λ_j, thermal conductivity of fiber material in the j-th row; λ_{ef}, effective thermal conductivity of composite material; T, temperature in matrix; t_j, temperature in fiber of the j-th row; q_z, heat-flux component parallel to axis Oz; A, B, Γ, constants.

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